$U(N)$ Gauged $\mathcal{N} = 2$ Supergravity and Partial Breaking of Local $\mathcal{N} = 2$ Supersymmetry

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Abstract

We study a $U(N)$ gauged $\mathcal{N} = 2$ supergravity model with one hypermultiplet parametrizing $SO(4,1)/SO(4)$ quaternionic manifold. Local $\mathcal{N} = 2$ supersymmetry is known to be spontaneously broken to $\mathcal{N} = 1$ in the Higgs phase of $U(1)_{graviphoton} \times U(1)$. Several properties are obtained of this model in the vacuum of unbroken $SU(N)$ gauge group. In particular, we derive mass spectrum analogous to the rigid counterpart and put the entire resulting potential on this vacuum in the standard superpotential form of $\mathcal{N} = 1$ supergravity.

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I.   Introduction

For more than a decade, $\mathcal{N} = 2$ supersymmetry both in its local and rigid realizations has played an important role in the theoretical developments of quantum field theory and particle physics. It has led us to the subject of exactly determined low energy effective actions [1, 2] and has inspired the construction of Lagrangians based on special Kähler geometry [3, 4, 5, 6, 7, 8, 9, 10]. These achievements have proven valuable in order to analyze some of the phenomena which occur in string theory.

Spontaneous breaking of $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ is an interesting problem in the light of its implications of string theory to the low energy $\mathcal{N} = 1$ supersymmety, which is phenomenologically promising. We give here a partial list of the references of this subject on the linear realizations [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. In particular, spontaneous partial breaking of rigid $\mathcal{N} = 2$ supersymmetry in the $U(N)$ gauge model with or without hypermultiplets has been demonstrated recently [22, 23, 24, 25] under a generic breaking pattern of the $U(N)$ gauge symmetry. Several other properties of this model have been obtained. It should be emphasized that the partial breaking of rigid $\mathcal{N} = 2$ supersymmetry is realized here in the Coulomb phase of overall $U(1)$, the Nambu-Goldstone fermion being the superpartner of the massless photon and that both interact with the $SU(N)$ sector thanks to the non-Lie algebraic property of the prepotential.

There are already considerable differences between the rigid special Kähler geometry and its local counterpart and between the hyperkähler geometry and the quaternionic geometry as have been emphasized in the literatures [7, 8, 9]. This is bound to be reflected in the comparison of the vacuum analysis of a rigid $\mathcal{N} = 2$ effective action with its supergravity counterpart. This will be a thrust of the present paper. Spontaneous partial breaking of local $\mathcal{N} = 2$ supersymmetry has been studied in [11, 12, 13, 17, 18, 19, 20, 21]. It was noted from the beginning that both the Higgs and the super-Higgs mechanisms must take place simultaneously and that the vacuum must lie in the Higgs phase of $U(1)_{graviphoton} \times U(1)$. The tight structure of the spectrum produced by the mechanisms requires at least one hypermultiplet with two $U(1)$ translational isometries to be introduced in the models.

In this paper, we study some of the basic and yet unexplored properties of the $U(N)$ gauged $\mathcal{N} = 2$ supergravity model in which local $\mathcal{N} = 2$ supersymmetry is partially broken spontaneously. In particular, the holomorphic section of our model is chosen as a generic function which leads to the nontrivial scalar coupling terms and the scalar potential. (In reference [12], a simple form of the section has been adopted.) In the next section, we briefly review $U(N)$ gauged $\mathcal{N} = 2$ supergravity in four dimensions and consider the model which contains a $U(N)$ vector multiplet and a hypermultiplet parametrizing $SO(4, 1)/SO(4)$

quaternionic manifold. Because of the choice of the section we need careful consideration of the vacuum, which is done in section three. We consider and solve the vacuum conditions of the model under the assumption of unbroken $SU(N)$ gauge symmetry. The second vacuum condition, which is a variation of the potential with respect to the hypermultiplet scalar b^u, was not considered before and the super-Higgs mechanism can not operate without this one. Partial breaking of local $\mathcal{N} = 2$ supersymmetry is exhibited. In section four, we derive the mass spectrum of the model and interpret it in terms of $\mathcal{N} = 1$ on-shell supermultiplets. In section five, we construct the entire Lagrangian on this vacuum and express it in terms of two superpotentials which are related to each other by a simple relation (5.9). The resulting form conforms to the standard form of $\mathcal{N} = 1$ supergravity.

II. $U(N)$ Gauged $\mathcal{N} = 2$ Supergravity

The field contents of $U(N)$ Gauged $\mathcal{N} = 2$ Supergravity are summarized as follows:

- *supergravity multiplet*
  consisting of the vierbein e^i_μ $(i, \mu = 0, 1, 2, 3)$, two gravitini ψ^A_μ $(A = 1, 2)$ and the graviphoton A^0_μ. (The upper and the lower position of the index A represent left and right chirality respectively.)

- *vector multiplet*
  consisting of a gauge boson A^a_μ, two gaugini λ^{aA} and a complex scalar z^a. The index a $(a = 1, \ldots, N^2)$ labels the generators of the $U(N)$ gauge group and $a = N^2 \equiv n$ refers to the overall $U(1)$. (The notation on the chirality is opposite to that of the gravitini, namely, the upper and the lower position denote right and left chirality respectively.)

- *hypermultiplet*
  consisting of two hyperini ζ^α $(\alpha = 1, 2)$ and four real scalars b^u $(u = 0, 1, 2, 3)$. (The upper and the lower position of the index α represent left and right chirality respectively.)

General construction of the Lagrangian of gauged $\mathcal{N} = 2$ supergravity has been given in [3, 5, 7]. We exhibit the parts of the Lagrangian and the supersymmetric transformation laws which are necessary for our analysis of the vacuum.

A. Vector Multiplet

The manifold associated with the vector multiplet is special Kähler of the local type [3, 4, 5, 6, 7, 8, 9, 10]. It is equipped with a holomorphic section,

$$\Omega(z) = \begin{pmatrix} X^\Lambda(z) \\ F_\Lambda(z) \end{pmatrix}, \quad \Lambda = 0, 1, \ldots, n \tag{2.1}$$

The index 0 refers to the graviphoton part. In terms of this section, the Kähler potential is given by,

$$\mathcal{K} = -\log i \langle \Omega | \bar{\Omega} \rangle = -\log i (\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda), \tag{2.2}$$

where

$$i \langle \Omega | \bar{\Omega} \rangle \equiv -i\Omega^T \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \Omega^*. \tag{2.3}$$

The non-holomorphic section is introduced by

$$V = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} \equiv e^{\mathcal{K}/2}\Omega = e^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}, \tag{2.4}$$

and its covariant derivative is

$$U_a \equiv \nabla_a V = (\partial_a + \tfrac{1}{2}\partial_a \mathcal{K})V \equiv \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda|a} \end{pmatrix}. \tag{2.5}$$

One characteristic property of $\mathcal{N} = 2$ supergravity, which follows from the special Kähler geometry of the local type, is the existence of totally symmetric rank-three tensor C_{abc} such that

$$\nabla_a U_b = iC_{abc}g^{cd^*}\bar{U}_{d^*}. \tag{2.6}$$

(See, for example, [7, 8]). The generalized gauge coupling matrix $\bar{\mathcal{N}}_{\Lambda\Sigma}$ is introduced via the following relations:

$$\bar{M}_\Lambda = \bar{\mathcal{N}}_{\Lambda\Sigma}\bar{L}^\Sigma, \quad h_{\Lambda|a} = \bar{\mathcal{N}}_{\Lambda\Sigma}f_a^\Sigma. \tag{2.7}$$

The solution is given in terms of two $(n+1) \times (n+1)$ matrices

$$f_I^\Lambda = \begin{pmatrix} f_a^\Lambda \\ \bar{L}^\Lambda \end{pmatrix}, \quad h_{\Lambda|I} = \begin{pmatrix} h_{\Lambda|a} \\ \bar{M}_\Lambda \end{pmatrix} \tag{2.8}$$

as

$$\bar{\mathcal{N}}_{\Lambda\Sigma} = h_{\Lambda|I}(f^{-1})_\Sigma^I. \tag{2.9}$$

It is well-known that this quantity appears in the kinetic term of the gauge bosons.

To specify the model, we need to choose the holomorphic section. Our choice, which is essentially that of [13], is

$$X^0(z) = \frac{1}{\sqrt{2}}, \quad F_0(z) = \frac{1}{\sqrt{2}} \left(2\mathcal{F}(z) - z^a \frac{\partial \mathcal{F}(z)}{\partial z^a} \right),$$

$$X^{\hat{a}}(z) = \frac{1}{\sqrt{2}} z^{\hat{a}}, \quad F_{\hat{a}}(z) = \frac{1}{\sqrt{2}} \frac{\partial \mathcal{F}(z)}{\partial z^{\hat{a}}}, \tag{2.10}$$

$$X^n(z) = \frac{1}{\sqrt{2}} \frac{\partial \mathcal{F}(z)}{\partial z^n}, \quad F_n(z) = -\frac{1}{\sqrt{2}} z^n,$$

where the index $\hat{a} = 1, \ldots, n-1$, labels the generators of $SU(N)$ subgroup. It has been obtained from the derivatives of the holomorphic function $F(X^0, X^a) = (X^0)^2 \mathcal{F}(X^a/X^0)$, that is, $\partial F/\partial X^\Lambda$ and performing the symplectic transformation $X^n \to -F_n, F_n \to X^n$. The Kähler potential and its derivatives are given by

$$\mathcal{K} = -\log \mathcal{K}_0, \tag{2.11}$$

$$\partial_a \mathcal{K}_0 = \frac{i}{2} \left(\mathcal{F}_a - \bar{\mathcal{F}}_a - (z^c - \bar{z}^c)\mathcal{F}_{ac} \right), \tag{2.12}$$

$$g_{ab^*} = \partial_a \partial_{b^*} \mathcal{K}$$

$$= \partial_a \mathcal{K} \partial_{b^*} \mathcal{K} - \frac{i}{2\mathcal{K}_0}(\mathcal{F}_{ab} - \bar{\mathcal{F}}_{ab}), \tag{2.13}$$

where $\mathcal{F}_a = \partial \mathcal{F}/\partial z^a$ and

$$\mathcal{K}_0 = i \left(\mathcal{F} - \bar{\mathcal{F}} - \frac{1}{2}(z^a - \bar{z}^a)(\mathcal{F}_a + \bar{\mathcal{F}}_a) \right). \tag{2.14}$$

Furthermore, the covariant derivative of f_a^Λ is

$$\nabla_a f_b^0 \equiv \frac{ie^{\mathcal{K}/2}}{\sqrt{2}} C_{abc} g^{cd^*} \partial_{d^*} \mathcal{K}$$

$$\equiv \partial_a f_b^0 + \Gamma_{ab}^c f_c^0 + \frac{1}{2}\partial_a \mathcal{K} f_b^0$$

$$= \frac{e^{\mathcal{K}/2}}{\sqrt{2}} \left(\partial_a \partial_b \mathcal{K} - \partial_a \mathcal{K} \partial_b \mathcal{K} + \frac{1}{\mathcal{K}_0} g^{cd^*} (\partial_a \partial_b \mathcal{K}_0 \partial_{d^*} \mathcal{K} + \partial_a \partial_b \partial_{d^*} \mathcal{K}_0)\partial_c \mathcal{K} \right), \tag{2.15}$$

$$\nabla_a f_b^n \equiv \frac{ie^{\mathcal{K}/2}}{\sqrt{2}} C_{abc} g^{cd^*} (\bar{\mathcal{F}}_{nd} + \partial_{d^*}\mathcal{K}\bar{\mathcal{F}}_n)$$

$$= \frac{e^{\mathcal{K}/2}}{\sqrt{2}} (\mathcal{F}_{nab} - \partial_a \mathcal{K} \partial_b \mathcal{K} \mathcal{F}_n + \partial_a \partial_b \mathcal{K} \mathcal{F}_n)$$

$$+ \frac{e^{\mathcal{K}/2}}{\sqrt{2}\mathcal{K}_0} g^{cd^*} (\partial_a \partial_b \mathcal{K}_0 \partial_{d^*}\mathcal{K} + \partial_a \partial_b \partial_{d^*}\mathcal{K}_0) (\mathcal{F}_{nc} + \partial_c \mathcal{K}\mathcal{F}_n). \tag{2.16}$$

5

The Christoffel symbol is defined as $\Gamma^c_{ab} = -g^{cd^*}\partial_b g_{ad^*}$. These equations will be used in the analysis of the potential term.

In order gauge the vector multiplet, first introduce the Killing vectors which are defined by

$$k^c_a \partial_c = f^c_{ab}z^b\partial_c, \qquad k^{c^*}_a \bar{\partial}_{c^*} = f^c_{ab}\bar{z}^{b^*}\bar{\partial}_{c^*}, \tag{2.17}$$

where f^a_{bc} is the structure constant of the $U(N)$ gauge group satisfying

$$[t_a, t_b] = if^c_{ab}t_c. \tag{2.18}$$

We will deal with the case in which the Lie derivative $\mathcal{L}_\Lambda$ satisfies

$$0 = \mathcal{L}_\Lambda \mathcal{K} = k^b_\Lambda \partial_b \mathcal{K} + k^{b^*}_\Lambda \partial_{b^*} \mathcal{K}. \tag{2.19}$$

The covariant derivative of the scalar fields, for example, takes the standard form:

$$\begin{aligned}
\nabla_\mu z^a &= \partial_\mu z^a + A^\Lambda_\mu k^a_\Lambda \\
&= \partial_\mu z^a + f^a_{bc}A^b_\mu z^c.
\end{aligned} \tag{2.20}$$

B. Hypermultiplet

Four real scalar components b^u of the hypermultiplet span the quaternionic manifold which is taken to be $SO(4,1)/SO(4)$. The quaternionic geometry is in general determined by a triplet of quaternionic potentials,

$$\begin{aligned}
\Omega^x &= \Omega^x_{uv}db^u \wedge db^v \\
&= d\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z, \qquad x = 1,2,3,
\end{aligned} \tag{2.21}$$

where $\omega^x = \omega^x_u db^u$ are the $SU(2)$ connections. In this paper, we take the same parametrizations as that of [12, 13]. The above quantities read

$$\omega^x_u = \frac{1}{b^0}\delta^x_u, \qquad \Omega^x_{0u} = -\frac{1}{2(b^0)^2}\delta^x_u, \qquad \Omega^x_{yz} = \frac{1}{2(b^0)^2}\epsilon^{xyz}. \tag{2.22}$$

The metric h_{uv} of this manifold is

$$h_{uv} = \frac{1}{2(b^0)^2}\delta_{uv}, \tag{2.23}$$

while the symplectic vielbein is

$$\mathcal{U}^{\alpha A}_u db^u, (\alpha, A = 1,2), \ \mathcal{U}^{\alpha A} = \frac{1}{2b^0}\epsilon^{\alpha\beta}(db^0 - i\sigma^x db^x)_\beta{}^A, \tag{2.24}$$

6

where σ^x are the standard Pauli matrices.

Let us introduce the Killing vectors k_Λ^u and the momentum maps $\mathcal{P}_\Lambda^x$ associated with two $U(1)$ translational isometries of this quaternionic manifold [13]:

$$
\begin{aligned}
k_0^u &= g_1 \delta^{u3} + g_2 \delta^{u2}, \quad k_{\hat{a}}^u = 0, \quad k_n^u = g_3 \delta^{u2}, \\
\mathcal{P}_0^x &= \frac{1}{b^0}(g_1 \delta^{x3} + g_2 \delta^{x2}), \quad \mathcal{P}_{\hat{a}}^x = 0, \quad \mathcal{P}_n^x = \frac{1}{b^0} g_3 \delta^{x2}.
\end{aligned}
\tag{2.25}
$$

Here $g_1, g_2, g_3, \in \mathbb{R}$ are coupling constants. These constants play the same role as the super-potential and the Fayet-Iliopoulos term do in the rigid theory [14, 22, 24, 25].

C. The Lagrangian of $\mathcal{N} = 2$ Supergravity

Let us write the parts of the Lagrangian of the $\mathcal{N} = 2$ gauged supergravity which is needed in our analysis:

$$
\mathcal{L} = \sqrt{-g} \left(\mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} - V(z, \bar{z}, b) + \ldots \right),
\tag{2.26}
$$

where

$$
\begin{aligned}
\mathcal{L}_{\text{kin}} &= R + g_{ab^*} \nabla_\mu z^a \nabla^\mu \bar{z}^{b^*} + h_{uv} \nabla_\mu b^u \nabla^\mu b^v + \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} (\bar{\psi}_\mu^A \gamma_\nu \nabla_\lambda \psi_{A\sigma} - \bar{\psi}_{A\mu} \gamma_\nu \nabla_\lambda \psi_\sigma^A) \\
&\quad + \frac{1}{4} (\text{Im} \mathcal{N})_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{4} (\text{Re} \mathcal{N})_{\Lambda\Sigma} F_{\mu\nu}^\Lambda \widetilde{F}^{\Sigma\mu\nu} \\
&\quad - i g_{ab^*} \bar{\lambda}^{aA} \gamma_\mu \nabla^\mu \lambda_A^{b^*} - 2i \bar{\zeta}^\alpha \gamma_\mu \nabla^\mu \zeta_\alpha + \ldots,
\end{aligned}
\tag{2.27}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= 2 S_{AB} \bar{\psi}_\mu^A \gamma^{\mu\nu} \psi_\nu^B + i g_{ab^*} W^{aAB} \bar{\lambda}_A^{b^*} \gamma_\mu \psi_B^\mu + 2i N_\alpha^A \bar{\zeta}^\alpha \gamma_\mu \psi_A^\mu \\
&\quad + \mathcal{M}^{\alpha\beta} \bar{\zeta}_\alpha \zeta_\beta + \mathcal{M}_{aB}^\alpha \bar{\zeta}_\alpha \lambda^{aB} + \mathcal{M}_{aA|bB} \bar{\lambda}_A^a \lambda^{bB} + h.c.,
\end{aligned}
\tag{2.28}
$$

$$
V(z, \bar{z}, b) = g_{ab^*} k_\Lambda^a k_\Sigma^{b^*} \bar{L}^\Lambda L^\Sigma + g^{ab^*} f_a^\Lambda \bar{f}_{b^*}^\Sigma \mathcal{P}_\Lambda^x \mathcal{P}_\Sigma^x + 4 h_{uv} k_\Lambda^u k_\Sigma^v \bar{L}^\Lambda L^\Sigma - 3 \bar{L}^\Lambda L^\Sigma \mathcal{P}_\Lambda^x \mathcal{P}_\Sigma^x.
\tag{2.29}
$$

Here $F_{\mu\nu}^\Lambda$ are the field strengths of the $U(N)$ gauge fields and that of the graviphoton field, and $\tilde{F}_{\mu\nu}^\Lambda$ are their Hodge duals. The supersymmetry transformation laws of the fermions are

$$
\delta \psi_{A\mu} = i S_{AB} \gamma_\mu \epsilon^B + \ldots,
\tag{2.30}
$$

$$
\delta \lambda^{aA} = W^{aAB} \epsilon_B + \ldots,
\tag{2.31}
$$

$$
\delta \zeta_\alpha = N_\alpha^A \epsilon_A + \ldots.
\tag{2.32}
$$

7

The matrices appearing in the supersymmetry transformation laws and in eq. (2.28) are composed of the geometric quantities listed in the last two subsections:

$$S_{AB} = \frac{i}{2}(\sigma_x)_{AB}\mathcal{P}^x_\Lambda L^\Lambda, \tag{2.33}$$

$$W^{aAB} = \epsilon^{AB}k^a_\Lambda \bar{L}^\Lambda + i(\sigma_x)^{AB}\mathcal{P}^x_\Lambda g^{ab^*}\bar{f}^\Lambda_{b^*} \equiv W^{aAB}_1 + W^{aAB}_2 \tag{2.34}$$

$$N^A_\alpha = 2\,\mathcal{U}^A_{\alpha u}k^u_\Lambda \bar{L}^\Lambda, \tag{2.35}$$

$$\mathcal{M}^{\alpha\beta} = -\mathcal{U}^{A\alpha}_u \mathcal{U}^{B\beta}_v \epsilon_{AB}\nabla^{[u}k^{v]}_\Lambda L^\Lambda, \tag{2.36}$$

$$\mathcal{M}^\alpha_{bB} = -4\,\mathcal{U}^\alpha_{Bu}k^u_\Lambda f^\Lambda_b, \tag{2.37}$$

$$\mathcal{M}_{aA|bB} = \frac{1}{2}\left(\epsilon_{AB}g_{ac^*}k^{c^*}_\Lambda f^\Lambda_b + i(\sigma_x)_{AB}\mathcal{P}^x_\Lambda \nabla_b f^\Lambda_a\right) \tag{2.38}$$

$$\equiv \mathcal{M}^1_{aA|bB} + \mathcal{M}^2_{aA|bB}. \tag{2.39}$$

We obtain explicit forms of these matrices from (2.11)-(2.13), (2.17) and (2.22)-(2.25):

$$S_{AB} = -\frac{ie^{\mathcal{K}/2}}{2\sqrt{2}b^0}\begin{pmatrix} i(g_2 + g_3\mathcal{F}_n) & g_1 \\ g_1 & i(g_2 + g_3\mathcal{F}_n) \end{pmatrix}, \tag{2.40}$$

$$W^{aAB}_1 = -ie^{\mathcal{K}/2}\mathcal{D}^a\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.41}$$

$$W^{aAB}_2 = \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0}g^{ab^*}\begin{pmatrix} g_2\partial_{b^*}\mathcal{K}+g_3(\bar{\mathcal{F}}_{nb}+\partial_{b^*}\mathcal{K}\bar{\mathcal{F}}_n) & ig_1\partial_{b^*}\mathcal{K} \\ ig_1\partial_{b^*}\mathcal{K} & g_2\partial_{b^*}\mathcal{K}+g_3(\bar{\mathcal{F}}_{nb}+\partial_{b^*}\mathcal{K}\bar{\mathcal{F}}_n) \end{pmatrix}, \tag{2.42}$$

$$N^A_\alpha = \frac{ie^{\mathcal{K}/2}}{\sqrt{2}b^0}\begin{pmatrix} g_1 & -i(g_2 + g_3\bar{\mathcal{F}}_n) \\ i(g_2 + g_3\bar{\mathcal{F}}_n) & -g_1 \end{pmatrix}, \tag{2.43}$$

$$\mathcal{M}^{\alpha\beta} = \frac{ie^{\mathcal{K}/2}}{\sqrt{2}b^0}\begin{pmatrix} -i(g_2 + g_3\mathcal{F}_n) & g_1 \\ g_1 & -i(g_2 + g_3\mathcal{F}_n) \end{pmatrix}, \tag{2.44}$$

$$\mathcal{M}^\alpha_{bB} = -\frac{\sqrt{2}ie^{\mathcal{K}/2}}{b^0}\begin{pmatrix} g_1\partial_a\mathcal{K} & i(g_2\partial_a\mathcal{K}+g_3(\mathcal{F}_{na}+\partial_a\mathcal{K}\mathcal{F}_n)) \\ -i(g_2\partial_a\mathcal{K}+g_3(\mathcal{F}_{nb}+\partial_a\mathcal{K}\mathcal{F}_n)) & g_1\partial_a\mathcal{K} \end{pmatrix}, \tag{2.45}$$

$$\mathcal{M}_{1;aA|bB} = -\frac{ie^{\mathcal{K}/2}}{2}g_{ac^*}(\partial_b + \partial_b\mathcal{K})\mathcal{D}^c\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.46}$$

$$\mathcal{M}_{2;aA|bB} = -\frac{1}{2b^0}\begin{pmatrix} g_2\nabla_b f^0_a+g_3\nabla_b f^n_a & -ig_1\nabla_b f^0_a \\ -ig_1\nabla_b f^0_a & g_2\nabla_b f^0_a+g_3\nabla_b f^n_a \end{pmatrix}$$

$$= \frac{ie^{\mathcal{K}/2}}{2\sqrt{2}}C_{abc}g^{cd^*}\begin{pmatrix} g_2\partial_{b^*}\mathcal{K}+g_3(\bar{\mathcal{F}}_{nb}+\partial_{b^*}\mathcal{K}\bar{\mathcal{F}}_n) & -ig_1\partial_{b^*}\mathcal{K} \\ -ig_1\partial_{b^*}\mathcal{K} & g_2\partial_{b^*}\mathcal{K}+g_3(\bar{\mathcal{F}}_{nb}+\partial_{b^*}\mathcal{K}\bar{\mathcal{F}}_n) \end{pmatrix}. \tag{2.47}$$

Here we have introduced

$$\mathcal{D}^a \;=\; \frac{i}{\sqrt{2}} f^a_{\ bc}\bar{z}^{b^*} z^c. \tag{2.48}$$

III.   Partial Breaking of $\mathcal{N} = 2$ Local Supersymmetry

By the gauging of hypermultiplet, the scalar potential takes a nontrivial form and is given by

$$V(z, \bar{z}, b) \;=\; e^{\mathcal{K}} g_{ab^*}\mathcal{D}^a\mathcal{D}^b + \frac{e^{\mathcal{K}}}{(b^0)^2} g^{ab^*} D^x_a \bar{D}^x_{b^*}$$

$$-\frac{e^{\mathcal{K}}}{2(b^0)^2}(\mathcal{E}^x + \mathcal{M}^x\mathcal{F}_n)(\mathcal{E}^x + \mathcal{M}^x\bar{\mathcal{F}}_n), \tag{3.1}$$

with

$$D^x_a \;=\; \frac{1}{\sqrt{2}}\left(\mathcal{E}^x\partial_a\mathcal{K} + \mathcal{M}^x(\mathcal{F}_{na} + \partial_a\mathcal{K}\mathcal{F}_n)\right), \tag{3.2}$$

$$\mathcal{E}^x \;=\; (0,\ g_2,\ g_1),$$

$$\mathcal{M}^x \;=\; (0,\ g_3,\ 0).$$

The first term comes from the $U(N)$ gauging of the vector multiplet while the second and the last terms correspond to gauging of the hypermultiplet.

Let us find the conditions which determine the minimum of the potential. Let us first consider the variations of V with respect to z^a. The derivative of the second and the third terms of V reads

$$\frac{e^{\mathcal{K}}}{(b^0)^2}\left((\partial_a\mathcal{K})g^{bc^*}D^x_b\bar{D}^x_{c^*} + (\partial_a g^{bc^*})D^x_b\bar{D}^x_{c^*} + g^{bc^*}(\partial_a D^x_b)\bar{D}^x_{c^*}\right)$$

$$= \frac{e^{\mathcal{K}}}{(b^0)^2}g^{bc^*}\bar{D}^x_{c^*}\left(\partial_a D^x_b - (\partial_b\mathcal{K})D^x_a + \frac{1}{\mathcal{K}_0}g^{ed^*}(\partial_a\partial_b\mathcal{K}_0\partial_{d^*}\mathcal{K} + \partial_a\partial_b\partial_{d^*}\mathcal{K}_0)D^x_e\right)$$

$$= \frac{ie^{\mathcal{K}}}{(b^0)^2}C_{abc}g^{bd^*}\bar{D}^x_{d^*}g^{ce^*}\bar{D}^x_{e^*}, \tag{3.3}$$

where we have used (2.15),(2.16) in the last equality. Thus, the first vacuum condition is

$$\langle\partial_c V\rangle = \langle\partial_c\left(e^{\mathcal{K}}g_{ab^*}\mathcal{D}^a\mathcal{D}^b\right)\rangle + \langle\frac{e^{\mathcal{K}}}{(b^0)^2}iC_{acd}g^{ab^*}\bar{D}^x_{b^*}g^{de^*}\bar{D}^x_{e^*}\rangle = 0, \tag{3.4}$$

The second vacuum condition is to be with respect to the hypermultiplet scalar b^u. As the potential contains only b^0, the condition reads

$$\langle\frac{\partial V}{\partial b^0}\rangle \;=\; -\frac{e^{\mathcal{K}}}{(b^0)^3}\langle 2g^{ab^*}D^x_a\bar{D}^x_{b^*} - (\mathcal{E}^x + \mathcal{M}^x\mathcal{F}_n)(\mathcal{E}^x + \mathcal{M}^x\bar{\mathcal{F}}_n)\rangle = 0. \tag{3.5}$$

As we search for the vacua with unbroken $SU(N)$ gauge symmetry in this paper, we will work on the condition $\langle z^a \rangle = \delta^{an}\lambda$. Then $\langle \mathcal{D}^a \rangle = \langle \frac{i}{\sqrt{2}} f^a{}_{bc} \bar{z}^{b*} z^c \rangle = 0$ holds and $\langle \partial_c (e^{\mathcal{K}} g_{ab*} \mathcal{D}^a \mathcal{D}^b) \rangle = 0$. For concreteness, we assume a form of the gauge invariant function $\mathcal{F}(z)$ as the one which parallels that of [24]:

$$\mathcal{F}(z) = -\frac{iC}{2}(z^n)^2 + \mathcal{G}(z), \tag{3.6}$$

$$\mathcal{G}(z) = \sum_{l=0}^{k} \frac{C_l}{l!} tr(z^a t_a)^l, \tag{3.7}$$

where $C \in \mathbb{R}$ and C_l are constant. We will see that C must be nonvanishing in order for the inverse of the Kähler metric to exist.

Let us compute the expectation value of the derivative of $\mathcal{F}$

$$\langle \mathcal{F}_a \rangle = \delta_{an} \langle \mathcal{F}_n \rangle,$$
$$\langle \mathcal{F}_{na} \rangle = \delta_{an} \langle \mathcal{F}_{nn} \rangle,$$
$$\langle \mathcal{F}_{\hat{a}\hat{b}} \rangle = \delta_{\hat{a}\hat{b}} \langle \mathcal{F}_{nn} - iC \rangle,$$
$$\langle \mathcal{F}_{nab} \rangle = \delta_{ab} \langle \mathcal{F}_{nnn} \rangle, \tag{3.8}$$

where the explicit form of $\langle \mathcal{F}_n \rangle$ and that of $\langle \mathcal{F}_{nn} \rangle$ are respectively

$$\langle \mathcal{F}_n \rangle = \sum_l \frac{C_l}{(l-1)!} \lambda^{l-1} + iC\lambda,$$

$$\langle \mathcal{F}_{nn} \rangle = \sum_l \frac{C_l}{(l-2)!} \lambda^{l-2}. \tag{3.9}$$

It is easy to compute $\partial_a \mathcal{K}$,

$$\langle \partial_a \mathcal{K} \rangle = -\frac{i\langle e^{\mathcal{K}} \rangle}{2} \langle \mathcal{F}_a - \bar{\mathcal{F}}_a - (\lambda - \bar{\lambda})\mathcal{F}_{an} \rangle$$
$$= \delta_{an} \langle \partial_n \mathcal{K} \rangle. \tag{3.10}$$

The Kähler metric g_{ab} is

$$\langle g_{ab*} \rangle = \begin{pmatrix} \langle g_{11*} \rangle & & & & \\ & \langle g_{11*} \rangle & & 0 & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \langle g_{nn} \rangle \end{pmatrix}, \tag{3.11}$$

10

with

$$\langle g_{11*} \rangle = -\frac{i\langle e^{\mathcal{K}} \rangle}{2} \langle \mathcal{F}_{nn} - \bar{\mathcal{F}}_{nn} - 2iC \rangle,$$

$$\langle g_{nn} \rangle = |\langle \partial_n \mathcal{K} \rangle|^2 - \frac{i\langle e^{\mathcal{K}} \rangle}{2} \langle \mathcal{F}_{nn} - \bar{\mathcal{F}}_{nn} \rangle. \tag{3.12}$$

Note that the diagonal components except $\langle g_{nn} \rangle$ take the same value. By substituting the above values, $\langle D_a^x \rangle$ and $\langle C_{abc} \rangle$ take the following expression:

$$\langle D_a^x \rangle = \delta_{an} \frac{1}{\sqrt{2}} \langle \mathcal{E}^x \partial_n \mathcal{K} + \mathcal{M}^x (\mathcal{F}_{nn} + \partial_n \mathcal{K} \mathcal{F}_n) \rangle,$$

$$= \delta_{an} \langle D_n^x \rangle$$

$$\langle C_{abc} \rangle = \frac{\langle e^{\mathcal{K}} \rangle}{2} \langle \mathcal{F}_{abc} \rangle. \tag{3.13}$$

Now we are ready to analyze (3.4) and (3.5). Substituting (3.8)-(3.13) into (3.4), we obtain

$$\langle \frac{ie^{2\mathcal{K}}}{2(b^0)^2} \mathcal{F}_{nnn} g^{nn^*} \bar{D}_{n^*}^x g^{nn^*} \bar{D}_{n^*}^x \rangle = 0. \tag{3.14}$$

The points $\langle \mathcal{F}_{nnn} \rangle = 0$ are unstable vacua because $\langle \partial_a \partial_{b^*} V \rangle = 0$. The points which satisfy $\langle g^{nn^*} \rangle = 0$ and $\langle \partial_n \mathcal{K} \rangle = 0$ are not stable. The vacuum condition reduces to

$$\langle \bar{D}_{n^*}^x \bar{D}_{n^*}^x \rangle = 0, \tag{3.15}$$

which implies

$$\left\langle\!\!\left\langle \mathcal{F}_n + \frac{\mathcal{F}_{nn}}{\partial_n \mathcal{K}} \right\rangle\!\!\right\rangle = -\left(\frac{g_2}{g_3} \pm i\frac{g_1}{g_3} \right). \tag{3.16}$$

where we use $\langle\!\langle \cdots \rangle\!\rangle$ for the vacuum expectation value of $\cdots$ which are determined as the solutions to (3.15). We have also assumed $g_3 \neq 0$. Note that if $g_3 = 0$ (3.15) leads to $g_1 = g_2 = 0$ and the supersymmetry is unbroken.

The second condition (3.5) reads

$$\left| g_1 \mp ig_3 \left\langle\!\!\left\langle \frac{\mathcal{F}_{nn}}{\partial_n \mathcal{K}} \right\rangle\!\!\right\rangle \right|^2 + g_1^2 - \langle\!\langle g^{nn^*} |\partial_n \mathcal{K}|^2 \rangle\!\rangle \, 2g_1^2 = 0. \tag{3.17}$$

When $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle = 0$, (3.12) imply $\langle\!\langle g^{nn^*} |\partial_n \mathcal{K}|^2 \rangle\!\rangle = 1$. Thus the above equality is satisfied. On the other hand, when $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle \neq 0$, (3.17) leads to $g_3 = 0$. This is proven in the appendix. $g_3 = 0$ conflicts with the assumption. The second vacuum condition thus reduces to

$$\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle = 0. \tag{3.18}$$

11

In the rigid theory [22, 24] with no hypermultiplet, there is no counterpart to this equation. In fact, we will see shortly that $\mathcal{N} = 2$ local supersymmetry is not broken partially without invoking the second vacuum condition. We conclude from (3.16) (3.18)

$$\langle\!\langle \mathcal{F}_n \rangle\!\rangle = -\left(\frac{g_2}{g_3} \pm i\frac{g_1}{g_3} \right). \tag{3.19}$$

In what follows, we take the $+$ sign. So $\partial_a \mathcal{K}$ and g_{ab^*} take the following expression:

$$\langle\!\langle \partial_a \mathcal{K} \rangle\!\rangle = -\delta_{an} \langle\!\langle e^{\mathcal{K}} \rangle\!\rangle \frac{g_1}{g_3},$$

$$\langle\!\langle g_{11^*} \rangle\!\rangle = -\langle\!\langle e^K \rangle\!\rangle C,$$

$$\langle\!\langle g_{nn^*} \rangle\!\rangle = |\langle\!\langle \partial_n \mathcal{K} \rangle\!\rangle|^2 = \langle\!\langle e^{2\mathcal{K}} \rangle\!\rangle \left(\frac{g_1}{g_3} \right)^2. \tag{3.20}$$

Note that $C \neq 0$ is necessary for the Kähler metric to be invertible.

Let us see if extended supersymmetries are spontaneously broken or not by considering the vacuum expectation values of the mass matrices (2.40)-(2.43). They are

$$\langle\!\langle S_{AB} \rangle\!\rangle = -\left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{2\sqrt{2}b^0} g_1 \right\rangle\!\!\right\rangle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$\langle\!\langle W^{aAB} \rangle\!\rangle = \delta^{an} \left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{\sqrt{2}b^0} (\partial_n \mathcal{K})^{-1} g_1 \right\rangle\!\!\right\rangle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$\langle\!\langle N_\alpha^A \rangle\!\rangle = \left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{\sqrt{2}b^0} g_1 \right\rangle\!\!\right\rangle \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}. \tag{3.21}$$

Notice that these matrices have zero eigenvalues. Introducing $\phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2)$ where $\phi \in \psi, \zeta, \lambda$, we obtain the vevs of (2.30), (2.31), (2.32).

$$\langle\!\langle \delta\psi_{+\mu} \rangle\!\rangle = \left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{2b^0} g_1 \right\rangle\!\!\right\rangle \gamma_\mu (\epsilon_1 + \epsilon_2)$$

$$\langle\!\langle \delta\lambda^{a+} \rangle\!\rangle = \delta^{an} \left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{b^0} g_1 (\partial_n \mathcal{K})^{-1} \right\rangle\!\!\right\rangle (\epsilon_1 + \epsilon_2)$$

$$\langle\!\langle \delta\zeta_- \rangle\!\rangle = \left\langle\!\!\left\langle \frac{ie^{\mathcal{K}/2}}{b^0} g_1 \right\rangle\!\!\right\rangle (\epsilon_1 + \epsilon_2)$$

$$\langle\!\langle \delta\psi_{-\mu} \rangle\!\rangle = \langle\!\langle \lambda^{a-} \rangle\!\rangle = \langle\!\langle \zeta_+ \rangle\!\rangle = 0. \tag{3.22}$$

Let us further introduce

$$\chi_\bullet \equiv \langle\!\langle \partial_n \mathcal{K} \rangle\!\rangle \lambda^{n+} + 2\zeta_-,$$

$$\eta_\bullet \equiv -\langle\!\langle \partial_n \mathcal{K} \rangle\!\rangle \lambda^{n+} + \zeta_-, \tag{3.23}$$

12

whose supersymmetry transformations are

$$\langle\!\langle \delta\chi_\bullet \rangle\!\rangle = \left\langle\!\!\left\langle \frac{3ie^{\mathcal{K}/2}}{b^0}g_1 \right\rangle\!\!\right\rangle (\epsilon_1 + \epsilon_2),$$

$$\langle\!\langle \delta\eta_\bullet \rangle\!\rangle = 0, \tag{3.24}$$

where the upper and lower position of dot represent left and right chirality respectively. As we see in the next section, gravitino ψ_-, hyperino ζ_+, gaugino λ^a_- and χ are massless fermions while gravitino ψ_+, gaugino $\lambda^{\hat{a}}_+$ and η are massive physical fermions. $\mathcal{N} = 2$ local supersymmetry is spontaneously broken to $\mathcal{N} = 1$ and χ is the Nambu-Goldstone fermion. We will confirm this in the next section.

IV. Mass Spectrum

A. Fermion Mass

Let us consider the fermion mass spectrum. Substituting (3.21) into $\mathcal{L}_{\text{Yukawa}}$, we obtain

$$\mathcal{L}_{\text{Yukawa}} = -i \left\langle\!\!\left\langle \frac{\sqrt{2}e^{\mathcal{K}/2}}{b^0}g_1 \right\rangle\!\!\right\rangle \left(\bar{\psi}^+_\mu\gamma^{\mu\nu}\psi^+_\nu - i\bar{\chi}^\bullet\gamma_\mu\psi^\mu_+ + \frac{1}{3}\bar{\chi}_\bullet\chi_\bullet - \frac{1}{3}\bar{\eta}_\bullet\eta_\bullet \right)$$

$$+ \frac{1}{2\sqrt{2}} \left\langle\!\!\left\langle \frac{e^{\mathcal{K}/2}}{b^0}g_3\mathcal{F}_{aan} \right\rangle\!\!\right\rangle \bar{\lambda}^{a-}\lambda^{a-} + \dots + h.c., \tag{4.1}$$

The Nambu-Goldstone fermion χ coupling to the gravitino ψ^+ can be removed from the action by redefining the gravitino:

$$\psi^+_\mu \rightarrow \psi^+_\mu + \frac{i}{6}\gamma_\mu\chi_\bullet. \tag{4.2}$$

We obtain

$$\mathcal{L}_{\text{Yukawa}} = -i \left\langle\!\!\left\langle \frac{\sqrt{2}e^{\mathcal{K}/2}}{b^0}g_1 \right\rangle\!\!\right\rangle \left(\bar{\psi}^+_\mu\gamma^{\mu\nu}\psi^+_\nu - \frac{1}{3}\bar{\eta}_\bullet\eta_\bullet \right) + \frac{1}{2\sqrt{2}}\sum_{a=1}^{n} \left\langle\!\!\left\langle \frac{e^{\mathcal{K}/2}}{b^0}g_3\mathcal{F}_{aan} \right\rangle\!\!\right\rangle \bar{\lambda}^{a-}\lambda^{a-} + h.c. . \tag{4.3}$$

The gravitino ψ_+ has acquired a mass by the super-Higgs mechanism.

The kinetic terms of the massive fermions are

$$\mathcal{L}^{(f)}_{\text{kin}} = \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}^A_\mu\gamma_\nu\partial_\lambda\psi_{A\sigma} - \frac{i}{3}\bar{\eta}^\bullet\gamma_\mu\partial^\mu\eta_\bullet - i\sum_{a=1}^{n} \langle\!\langle g_{aa^*} \rangle\!\rangle \bar{\lambda}^{a-}\gamma_\mu\partial^\mu\lambda^{a^*}_- + \dots + h.c. \tag{4.4}$$

13

We obtain the mass of the fermions from the equations of motion. The gravitino mass m and the mass of the gauginos m_a are respectively

$$m = \left| \left\langle\!\!\left\langle \frac{\sqrt{2}e^{\mathcal{K}/2}}{b^0} g_1 \right\rangle\!\!\right\rangle \right|,$$

$$m_a = \left| \left\langle\!\!\left\langle \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0} g_3 \mathcal{F}_{aan} g^{aa^*} \right\rangle\!\!\right\rangle \right|. \tag{4.5}$$

Notice that the mass of the physical fermion $\eta_\bullet$ is the same as the gravitino, that is, m. ψ_+ and $\eta_\bullet$ will form a $\mathcal{N} = 1$ massive multiplet of spin (3/2,1,1,1/2), while λ^{a-}, together with the scalar fields, will form $\mathcal{N} = 1$ massive chiral multiplet. This mass spectrum is analogous to the rigid counterpart [24].

B. Boson Mass

Let us compute the masses of the scalar fields by introducing the shifted fields $\tilde{z}^a = z^a - \langle\!\langle z^a \rangle\!\rangle$. The second derivatives can be easily evaluated

$$\langle\!\langle \partial_a \partial_{b*} V \rangle\!\rangle = \left\langle\!\!\left\langle \frac{2ie^{\mathcal{K}}}{(b^0)^2} C_{acd} g^{ce^*} (\partial_{b*} \bar{D}^x_{e*}) g^{df^*} \bar{D}^x_{f*} \right\rangle\!\!\right\rangle$$

$$= \delta_{ab} \left\langle\!\!\left\langle \frac{e^{\mathcal{K}}}{2(b^0)^2} |g_3 \mathcal{F}_{aan}|^2 g^{aa^*} \right\rangle\!\!\right\rangle,$$

$$\langle\!\langle \partial_a \partial_b V \rangle\!\rangle = 0. \tag{4.6}$$

Thus, the kinetic terms and the mass terms are

$$\sum_a \left(\langle\!\langle g_{aa^*} \rangle\!\rangle \, \partial_\mu \tilde{z}^a \partial^\mu \bar{\tilde{z}}^{a^*} - \left\langle\!\!\left\langle \frac{e^{\mathcal{K}}}{2(b^0)^2} |g_3 \mathcal{F}_{aan}|^2 g^{aa^*} \right\rangle\!\!\right\rangle \tilde{z}^a \bar{\tilde{z}}^{a^*} \right). \tag{4.7}$$

The mass of $\tilde{z}^a$ is the same as (4.5), namely, the mass of λ^{a-}. They form N^2 massive chiral multiplets as we have anticipated,

The gauge boson masses appear in the kinetic terms of the hypermultiplet scalars

$$h_{uv} \nabla_\mu b^u \nabla^\mu b^v = \frac{1}{2(b^0)^2} (g_1^2 A^0_\mu A^{0\mu} + g_3^2 A'^n_\mu A'^{n\mu}) + \dots, \tag{4.8}$$

where

$$A'^n_\mu = A^n_\mu + \left(\frac{g_2}{g_3} \right) A^0_\mu. \tag{4.9}$$

14

The kinetic terms of the gauge bosons are $\frac{1}{4}(\mathrm{Im}\,\mathcal{N})_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu}$, and we compute the generalized coupling matrix $\mathcal{N}$ (2.9) on the vacuum:

$$
\langle\!\langle\mathcal{N}_{\Lambda\Sigma}\rangle\!\rangle = \begin{pmatrix}
\langle\!\langle\mathcal{N}_{00}\rangle\!\rangle & 0 & \cdots & \cdots & 0 & \langle\!\langle\mathcal{N}_{0n}\rangle\!\rangle \\
0 & \langle\!\langle\mathcal{G}_{11}\rangle\!\rangle & 0 & \cdots & 0 & 0 \\
\vdots & 0 & \langle\!\langle\mathcal{G}_{22}\rangle\!\rangle & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots \\
0 & 0 & \cdots & 0 & \langle\!\langle\mathcal{G}_{n-1,n-1}\rangle\!\rangle & 0 \\
\langle\!\langle\mathcal{N}_{n0}\rangle\!\rangle & 0 & \cdots & \cdots & 0 & \langle\!\langle\mathcal{N}_{nn}\rangle\!\rangle
\end{pmatrix}, \tag{4.10}
$$

with

$$
\mathrm{Im}\,\langle\!\langle\mathcal{N}_{00}\rangle\!\rangle = \left\langle\!\left\langle\frac{e^{-\mathcal{K}}}{2}\right\rangle\!\right\rangle\frac{g_1^2 + g_3^2}{g_1^2},
$$

$$
\mathrm{Im}\,\langle\!\langle\mathcal{N}_{0n}\rangle\!\rangle = \mathrm{Im}\,\langle\!\langle\mathcal{N}_{n0}\rangle\!\rangle = \left\langle\!\left\langle\frac{e^{-\mathcal{K}}}{2}\right\rangle\!\right\rangle\frac{g_2 g_3}{g_1^2},
$$

$$
\mathrm{Im}\,\langle\!\langle\mathcal{N}_{nn}\rangle\!\rangle = \left\langle\!\left\langle\frac{e^{-\mathcal{K}}}{2}\right\rangle\!\right\rangle\left(\frac{g_3}{g_1}\right)^2. \tag{4.11}
$$

Therefore the gauge boson kinetic terms are

$$
\frac{1}{4}\mathrm{Im}\,\langle\!\langle\mathcal{N}_{\Lambda\Sigma}\rangle\!\rangle F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} = -\left\langle\!\left\langle\frac{e^{-\mathcal{K}}}{8}\right\rangle\!\right\rangle F^0_{\mu\nu}F^{0\mu\nu} - \left\langle\!\left\langle\frac{e^{-\mathcal{K}}}{8}\right\rangle\!\right\rangle\left(\frac{g_3}{g_1}\right)^2 F'^n_{\mu\nu}F'^{n\mu\nu}
$$

$$
+ \frac{1}{4}\sum_{\hat{a}}\mathrm{Im}\,\langle\!\langle\mathcal{G}_{\hat{a}\hat{a}}\rangle\!\rangle F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu}, \tag{4.12}
$$

where we have defined $F'^n_{\mu\nu} = \partial_\mu A'^n_\nu - \partial_\nu A'^n_\mu$. We can read off the masses of gauge boson A^0_μ and A'^n_μ from (4.8) and (4.12). Both of them agree with (4.5).

We summarize the mass spectrum of our model in the table below:

$\mathcal{N} = 1$ multiplet	field	mass
gravity multiplet	e^a_μ, ψ^-_μ	0
spin-3/2 multiplet	ψ^+_μ, A^0_μ, A'^n_μ, $\eta_\bullet$	m
$SU(N)$ vector multiplet	$A^{\hat{a}}_\mu$, $\lambda^{\hat{a}+}$	0
$SU(N)$ adjoint chiral multiplet	$\lambda^{\hat{a}-}$, $z^{\hat{a}}$	$m^{\hat{a}}$
chiral multiplet	λ^{n-}, z^n	m^n
chiral multiplet	ζ_+, b^0, b^1	0

The $\mathcal{N} = 1$ gravity multiplet consists of the vierbein and the gravitino ψ^-_μ while the massive gravitino ψ^+_μ, $U(1)$ gauge boson A'^n_μ, the graviphoton A^0_μ and the fermion $\eta_\bullet$ form a massive

15

spin-3/2 multiplet. The $\mathcal{N} = 2$ $U(N)$ vector multiplet is divided into a $\mathcal{N} = 1$ vector multiplet and a chiral multiplet. The $\mathcal{N} = 1$ $SU(N)$ vector multiplet consists of massless gauge bosons $A_\mu^{\hat{a}}$ and gauginos $\lambda^{\hat{a}+}$. On the other hand, the gaugino $\lambda^{\hat{a}-}$ and the scalar field $z^{\hat{a}}$ form chiral multiplets which belong to the $SU(N)$ adjoint representation. The hyperino ζ_+ and the scalars b^0, b^1 form an $\mathcal{N} = 1$ chiral multiplet.

Note that the $U(1)_{graviphoton} \times U(N)$ gauge symmetry is broken to $SU(N)$ and the vacuum lies in the Higgs phase.

V. $\mathcal{N} = 1$ Lagrangian

In the last section, we have considered the lowest order terms with respect to the fermion fields and the shifted scalar fields $\tilde{z}^a$ in $\mathcal{L}_{\text{Yukawa}}$ and V. We will now reexpress the remaining terms in $\mathcal{L}_{\text{Yukawa}}$ and V by $\tilde{z}^a$ as well. In [29, 30], the reduction procedure from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ has been completed and $\mathcal{L}_{\text{Yukawa}}$ and V have been given in terms of a superpotential. Here, we find that the resulting $\mathcal{N} = 1$ Lagrangian on the vacuum is written by the superpotentials which are related each other by eq. (5.9).

The holomorphic function $\mathcal{F}(z)$ is expanded in the shifted fields $\tilde{z}^a$ as,

$$\mathcal{F}(z) = \mathcal{F}(\langle\!\langle z \rangle\!\rangle + \tilde{z}) = \langle\!\langle \mathcal{F} \rangle\!\rangle + \tilde{\mathcal{F}}, \tag{5.1}$$

where

$$\tilde{\mathcal{F}} = \langle\!\langle \mathcal{F}_a \rangle\!\rangle \, \tilde{z}^a + \frac{1}{2!} \, \langle\!\langle \mathcal{F}_{ab} \rangle\!\rangle \, \tilde{z}^a \tilde{z}^b + \frac{1}{3!} \, \langle\!\langle \mathcal{F}_{abc} \rangle\!\rangle \, \tilde{z}^a \tilde{z}^b \tilde{z}^c + \dots. \tag{5.2}$$

Similarly, $\mathcal{F}_a$ and $\mathcal{F}_{ab}$ are

$$\mathcal{F}_a = \langle\!\langle \mathcal{F}_a \rangle\!\rangle + \langle\!\langle \mathcal{F}_{ab} \rangle\!\rangle \, \tilde{z}^b + \frac{1}{2!} \, \langle\!\langle \mathcal{F}_{abc} \rangle\!\rangle \, \tilde{z}^b \tilde{z}^c + \dots = \tilde{\mathcal{F}}_a, \, \mathcal{F}_{ab} = \langle\!\langle \mathcal{F}_{ab} \rangle\!\rangle + \langle\!\langle \mathcal{F}_{abc} \rangle\!\rangle \, \tilde{z}^c + \dots = \tilde{\mathcal{F}}_{ab}. \tag{5.3}$$

The derivatives are taken with respect to $\tilde{z}^a$ in $\tilde{\mathcal{F}}_a$ and $\tilde{\mathcal{F}}_{ab}$. The Kähler potential and its derivatives are

$$\mathcal{K} = -\log i \left[\langle\!\langle \mathcal{F} - \bar{\mathcal{F}} \rangle\!\rangle + \tilde{\mathcal{F}} - \bar{\tilde{\mathcal{F}}} - \frac{1}{2}(\langle\!\langle z^a - \bar{z}^a \rangle\!\rangle + z^a - \bar{z}^a)(\tilde{\mathcal{F}}_a + \bar{\tilde{\mathcal{F}}}_a) \right], \tag{5.4}$$

$$\partial_a \mathcal{K} = -\frac{i}{2\mathcal{K}_0}(\tilde{\mathcal{F}}_a - \bar{\tilde{\mathcal{F}}}_a - (\langle\!\langle z^a - \bar{z}^a \rangle\!\rangle + z^a - \bar{z}^a)\tilde{\mathcal{F}}_{ab}) = \tilde{\partial}_a \mathcal{K}, \tag{5.5}$$

$$g_{ab^*} = \tilde{\partial}_a \mathcal{K} \tilde{\partial}_{b^*} \mathcal{K} - \frac{i}{2\mathcal{K}_0}(\tilde{\mathcal{F}}_{ab} - \bar{\tilde{\mathcal{F}}}_{ab}) = \tilde{g}_{ab^*}, \tag{5.6}$$

where $\tilde{\partial}_a = \partial/\partial \tilde{z}^a$.

Let us now define the 'two' superpotentials by

$$\mathcal{W}(\tilde{z}, \bar{\tilde{z}}) \;\; \equiv \;\; e^{\mathcal{K}/2}W(\tilde{z}) \equiv 2(S_{11} - S_{12}) = \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0}g_3(\tilde{\mathcal{F}}_n - \langle\!\langle\mathcal{F}_n\rangle\!\rangle), \tag{5.7}$$

$$\mathcal{S}(\tilde{z}, \bar{\tilde{z}}) \;\; \equiv \;\; e^{\mathcal{K}/2}S(\tilde{z}) \equiv 2(S_{11} + S_{12}) = \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0}(2g_2 + g_3(\tilde{\mathcal{F}}_n + \langle\!\langle\mathcal{F}_n\rangle\!\rangle)), \tag{5.8}$$

where S_{AB} is the gravitino mass matrix. Note that $\mathcal{W}$ and $\mathcal{S}$ are related as follows:

$$\mathcal{W} = \mathcal{S} + i\frac{\sqrt{2}g_1 e^{\mathcal{K}/2}}{b^0}. \tag{5.9}$$

Thus, they are not independent quantities. In the following, however, we will write down the resulting Lagrangian, using both $\mathcal{W}$ and $\mathcal{S}$. These quantities appear in the gravitino mass terms as

$$2S_{AB}\bar{\psi}^A_\mu\gamma^{\mu\nu}\psi^B_\nu \;\; = \;\; \mathcal{W}\bar{\psi}^-_\mu\gamma^{\mu\nu}\psi^-_\nu + \mathcal{S}\bar{\psi}^+_\mu\gamma^{\mu\nu}\psi^+_\nu. \tag{5.10}$$

The covariant derivative of $\mathcal{W}$ and that of $\mathcal{S}$ are respectively

$$\tilde{\nabla}_a\mathcal{W} \;\; = \;\; \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0}g_3(\tilde{\mathcal{F}}_{na} + \tilde{\partial}_a\mathcal{K}\tilde{\mathcal{F}}_n - \tilde{\partial}_a\mathcal{K}\langle\!\langle\mathcal{F}_n\rangle\!\rangle)$$

$$= \;\; g_{ab^*}(\bar{W}^{b^*}_{2;11} - \bar{W}^{b^*}_{2;12}), \tag{5.11}$$

$$\tilde{\nabla}_a\mathcal{S} \;\; = \;\; \frac{e^{\mathcal{K}/2}}{\sqrt{2}b^0}(2g_2\tilde{\partial}_a\mathcal{K} + g_3(\tilde{\mathcal{F}}_{na} + \tilde{\partial}_a\mathcal{K}\tilde{\mathcal{F}}_n + \tilde{\partial}_a\mathcal{K}\langle\!\langle\mathcal{F}_n\rangle\!\rangle))$$

$$= \;\; g_{ab^*}(\bar{W}^{b^*}_{2;11} + \bar{W}^{b^*}_{2;12}), \tag{5.12}$$

$$\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{W} \;\; = \;\; \sqrt{2}(\mathcal{M}_{2;a1|b1} - \mathcal{M}_{2;a1|b2}), \tag{5.13}$$

$$\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{S} \;\; = \;\; \sqrt{2}(\mathcal{M}_{2;a1|b1} + \mathcal{M}_{2;a1|b2}), \tag{5.14}$$

where $\bar{W}^{b^*}_{AB} = (W^{bAB})^*$. These are used to evaluate the second term and the last term of $\mathcal{L}_{\text{Yukawa}}$:

$$ig_{ab^*}W^{aAB}\bar{\lambda}^{b^*}_A\gamma_\mu\psi^\mu_B \;\; = \;\; e^{\mathcal{K}/2}\tilde{g}_{ab^*}\mathcal{D}^a(\bar{\lambda}^{b^*}_-\gamma_\mu\psi^\mu_+ - \bar{\lambda}^{b^*}_+\gamma_\mu\psi^\mu_-)$$

$$+ \;\; i\bar{\tilde{\nabla}}_{a^*}\bar{\mathcal{W}}\bar{\lambda}^{b^*}_-\gamma_\mu\psi^\mu_- + i\bar{\tilde{\nabla}}_{a^*}\bar{\mathcal{S}}\bar{\lambda}^{b^*}_+\gamma_\mu\psi^\mu_+, \tag{5.15}$$

$$\mathcal{M}_{aA|bB}\bar{\lambda}^{aA}\lambda^{bB} \;\; = \;\; \mathcal{M}_{1;a1|b2}(\bar{\lambda}^{a-}\lambda^{b+} - \bar{\lambda}^{a+}\lambda^{b-})$$

$$+ \;\; \frac{1}{\sqrt{2}}\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{W}\bar{\lambda}^{a-}\lambda^{b-} + \frac{1}{\sqrt{2}}\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{S}\bar{\lambda}^{a+}\lambda^{b+}. \tag{5.16}$$

We now manage to reexpress $\mathcal{L}_{\text{Yukawa}}$ by the shifted scalar fields, the superpotentials and

their covariant derivatives:

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= \mathcal{W}\bar{\psi}_\mu^- \gamma^{\mu\nu}\psi_\nu^- + i(\bar{\tilde{\nabla}}_{a*}\mathcal{W}\bar{\lambda}_-^{a*} - 2\bar{\mathcal{W}}\bar{\zeta}^+)\gamma_\mu\psi_-^\mu - e^{\mathcal{K}/2}\tilde{g}_{ab*}\mathcal{D}^a\bar{\lambda}_+^{b*}\gamma_\mu\psi_-^\mu \\
&\quad + \mathcal{S}\bar{\psi}_\mu^+ \gamma^{\mu\nu}\psi_\nu^+ + i(\bar{\tilde{\nabla}}_{a*}\bar{\mathcal{S}}\bar{\lambda}_+^{a*} + 2\bar{\mathcal{S}}\bar{\zeta}^-)\gamma_\mu\psi_+^\mu + e^{\mathcal{K}/2}\tilde{g}_{ab*}\mathcal{D}^a\bar{\lambda}_-^{b*}\gamma_\mu\psi_+^\mu \\
&\quad + \mathcal{W}\bar{\zeta}_+\zeta_+ + \mathcal{S}\bar{\zeta}_-\zeta_- - 2\tilde{\nabla}_a\mathcal{W}\bar{\zeta}_+\lambda^{a-} + 2\tilde{\nabla}_a\mathcal{S}\bar{\zeta}_-\lambda^{a+} \qquad (5.17)\\
&\quad + \mathcal{M}_{1;a1|b2}(\bar{\lambda}^{a-}\lambda^{b+} - \bar{\lambda}^{a+}\lambda^{b-}) + \frac{1}{\sqrt{2}}\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{W}\bar{\lambda}^{a-}\lambda^{b-} + \frac{1}{\sqrt{2}}\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{S}\bar{\lambda}^{a+}\lambda^{b+} + h.c. \ .
\end{aligned}
$$

Let us turn to the scalar potential V, which we rewrite in terms of the mass matrices as

$$
V = -12\bar{S}^{1A}S_{A1} + \tilde{g}_{ab*}\bar{W}_{1A}^{b*}W^{a1A} + 2\bar{N}_1^\alpha N_\alpha^1. \qquad (5.18)
$$

This equation is also obtained from the supergravity Ward identities in the reference [27]. Note that $\bar{S}^{AB} = (S_{AB})^*$ and $\bar{N}_A^\alpha = (N_\alpha^A)^*$. The first term is

$$
\begin{aligned}
-12\bar{S}^{1A}S_{A1} &= -6((S_{11} - S_{12})(\bar{S}^{11} - \bar{S}^{12}) + (S_{11} + S_{12})(\bar{S}^{11} + \bar{S}^{12})) \\
&= -\frac{3}{2}(|\mathcal{W}|^2 + |\mathcal{S}|^2), \qquad (5.19)
\end{aligned}
$$

and the second term is

$$
\begin{aligned}
\tilde{g}_{ab*}\bar{W}_{1A}^{b*}W^{a1A} &= \tilde{g}_{ab*}\left(\bar{W}_{1;12}^{b*}W_1^{a12} + \bar{W}_{2;11}^{b*}W_2^{a11} + \bar{W}_{2;12}^{b*}W_2^{a12}\right) \\
&= e^{\mathcal{K}}\tilde{g}_{ab*}\mathcal{D}^a\mathcal{D}^b + \frac{1}{2}\tilde{g}^{ab*}\tilde{\nabla}_a\mathcal{W}\bar{\tilde{\nabla}}_a\bar{\mathcal{W}} + \frac{1}{2}\tilde{g}^{ab*}\tilde{\nabla}_a\mathcal{S}\bar{\tilde{\nabla}}_a\bar{\mathcal{S}}. \qquad (5.20)
\end{aligned}
$$

In the first equality, we have used (2.19). The last term is

$$
\begin{aligned}
2\bar{N}_1^\alpha N_\alpha^1 &= |\mathcal{W}|^2 + |\mathcal{S}|^2 \\
&= \frac{1}{2}h^{uv}\nabla_u\mathcal{W}\nabla_v\bar{\mathcal{W}} + \frac{1}{2}h^{uv}\nabla_u\mathcal{S}\nabla_v\bar{\mathcal{S}}, \qquad (5.21)
\end{aligned}
$$

where $u, v = 0, 1$ and $h_{uv} \equiv \delta_{uv}/2(b^0)^2$. Note that $\nabla_u\mathcal{W} = \partial_u\mathcal{W}$. Substituting (5.19)-(5.21) into (5.18), we obtain

$$
\begin{aligned}
V &= e^{\mathcal{K}/2}g_{ab*}\mathcal{D}^a\mathcal{D}^b + \frac{1}{2}\tilde{g}^{ab*}\tilde{\nabla}_a\mathcal{W}\bar{\tilde{\nabla}}_a\bar{\mathcal{W}} + \frac{1}{2}\tilde{g}^{ab*}\tilde{\nabla}_a\mathcal{S}\bar{\tilde{\nabla}}_a\bar{\mathcal{S}} \\
&\quad - \frac{3}{2}|\mathcal{W}|^2 - \frac{3}{2}|\mathcal{S}|^2 + \frac{1}{2}h^{uv}\nabla_u\mathcal{W}\nabla_v\bar{\mathcal{W}} + \frac{1}{2}h^{uv}\nabla_u\mathcal{S}\nabla_v\bar{\mathcal{S}}. \qquad (5.22)
\end{aligned}
$$

This is the final form of the scalar potential. We can see that $\mathcal{L}_{\text{Yukawa}}$ and the scalar potential take essentially the same form as that of the usual $\mathcal{N} = 1$ supergravity models (such as [28] or [29, 30]).

As is pointed out in [13], if we force the gravity and the hypermultiplet to decouple, the gravitino mass (4.5) becomes zero. Thus, the gauge boson corresponding to the overall $U(1)$ and the graviphoton become massless in this limit. The Higgs phase of overall $U(1)_{graviphoton} \times U(1)$ approaches the Coulomb phase.

VI.   Acknowledgements

Appendix

Here we prove the second vacuum condition (3.17) leads to $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle = 0$. As mentioned above, if $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle = 0$, (3.17) is automatically satisfied. Thus, let us consider the case $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle \neq 0$. We write $\mathcal{F}_{nn}$ as

$$\mathcal{F}_{nn} = F_1 + iF_2, \tag{A.1}$$

where $F_1, F_2 \in \mathbb{R}$. From (3.12), (3.10) and (3.16), by using F_1 and F_2, we obtain

$$\langle\!\langle g_{nn^*} \rangle\!\rangle = \langle\!\langle |\partial_n \mathcal{K}|^2 \rangle\!\rangle + \frac{F_2}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle}, \tag{A.2}$$

$$\langle\!\langle \partial_n \mathcal{K} + \partial_{n^*} \mathcal{K} \rangle\!\rangle = \frac{i}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle} \left(\left\langle\!\!\left\langle \frac{\mathcal{F}_{nn}}{\partial_n \mathcal{K}} - \frac{\bar{\mathcal{F}}_{nn}}{\partial_{n^*} \mathcal{K}} \right\rangle\!\!\right\rangle \pm 2i \frac{g_1}{g_3} + (\lambda - \bar{\lambda}) F_1 \right), \tag{A.3}$$

$$\langle\!\langle \partial_n \mathcal{K} - \partial_{n^*} \mathcal{K} \rangle\!\rangle = \frac{1}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle} (\lambda - \bar{\lambda}) F_2. \tag{A.4}$$

The condition (3.17) can be written as

$$0 = 2g_1^2 - 2g_1^2 \langle\!\langle g^{nn^*} |\partial_n \mathcal{K}|^2 \rangle\!\rangle \mp iY g_1 g_3 + g_3^2 \left\langle\!\!\left\langle |\frac{\mathcal{F}_{nn}}{\partial_n \mathcal{K}}|^2 \right\rangle\!\!\right\rangle, \tag{A.5}$$

where we have defined Y as

$$\begin{aligned} Y &\equiv \left\langle\!\!\left\langle \frac{\mathcal{F}_{nn}}{\partial_n \mathcal{K}} - \frac{\bar{\mathcal{F}}_{nn}}{\partial_{n^*} \mathcal{K}} \right\rangle\!\!\right\rangle \\ &= \frac{1}{\langle\!\langle |\partial_n \mathcal{K}|^2 \rangle\!\rangle} \left[F_1 \langle\!\langle \partial_n \mathcal{K} - \partial_{n^*} \mathcal{K} \rangle\!\rangle - \frac{F_2}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle} \left(Y \pm 2i \frac{g_1}{g_3} + (\lambda - \bar{\lambda}) F_1 \right) \right]. \end{aligned} \tag{A.6}$$

In the second equality, we have used (A.3). Using (A.2), we can solve the above equation for Y:

$$Y \langle\!\langle g_{nn^*} \rangle\!\rangle = \mp \frac{F_2}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle} 2i \frac{g_1}{g_3}. \tag{A.7}$$

Substituting (A.7) into (A.5), we get

$$
\begin{aligned}
0 &= 2g_1^2 - 2g_1^2 \left\langle\!\!\left\langle g^{nn^*}|\partial_n\mathcal{K}|^2 \right\rangle\!\!\right\rangle - 2g_1^2 \left\langle\!\!\left\langle g^{nn^*} \frac{F_2}{\langle\!\langle \mathcal{K}_0 \rangle\!\rangle} \right\rangle\!\!\right\rangle + g_3^2 \left\langle\!\!\left\langle |\frac{\mathcal{F}_{nn}}{\partial_n\mathcal{K}}|^2 \right\rangle\!\!\right\rangle \\
&= g_3^2 \left\langle\!\!\left\langle |\frac{\mathcal{F}_{nn}}{\partial_n\mathcal{K}}|^2 \right\rangle\!\!\right\rangle,
\end{aligned}
\tag{A.8}
$$

where we have used (A.2). Therefore, we conclude that when $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle \neq 0$, the vacuum condition leads to $g_3 = 0$. This conflicts the assumption which is written in below (3.16). Thus, we can say that the second vacuum condition implies $\langle\!\langle \mathcal{F}_{nn} \rangle\!\rangle = 0$.

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